Differential Equations

LAB 2

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**Question 1**

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As can be seen from the first quiver plot, one of the equilibrium points is -1, where initial values that get solutions above -1 decrease and below -1 increase.

f = @(t,y) (y+1)\*(y-1);

quiver244(f, -1, 0.5,-3,3,1,'r')





After rearranging the equation it can be seen from the second quiver plot, the second equilibrium point is 1, where initial values that get solutions above 1 increase and below 1 decrease.

f = @(t,y) (y-1)\*(y+1);

quiver244(f, -1, 0.5,-3,3,1,'r')



As can be seen from the sample plot, y=1 and y=-1 are the equilibrium points. Any solution above 1 will increase and below 1 will decrease so y=1 is unstable. While any solution above -1 decreases and below -1 increases so y=-1 is stable.

**Question 2**

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From the first quiver plot the equilibrium point is y=0.



From the second quiver plot the equilibrium point is y=2.





From the third quiver plot the equilibrium point is y=-4.



For y(0) between -6 and -4, the solution goes to -4 as t gets large.

For y(0) between -4 and 0, the solution goes to -4 as t gets large.

For y(0) between 0 and 2, the solution goes to 2 as t gets large.

For y(0) between 2 and 4, the solution goes to 4 as t gets large.”

**Question 3**

For each of the last two parts, characterize the stability of each of the equilibrium solutions. How does this correspond to the way it shows up in the differential equation?

Part 1

y=1, y=-1

y=1 🡪 unstable

y=-1 🡪 stable

The sign for the derivative below -1 is + while the sign above is –, so + to – is stable

The sign for the derivative below 1 is - while the sign above is +, so - to + is unstable

Part 2

y=2, y=0, y=-4

y=2 🡪 semi-stable

y=0 🡪 unstable

y=-4 🡪 stable

The sign for the derivative below 2 is + while the sign above is +, so + to + is semi-stable

The sign for the derivative below 0 is - while the sign above is +, so - to + is unstable

The sign for the derivative below -4 is + while the sign above is –, so + to – is stable

**Question 4**

function phaseLine(f, ymin, ymax);

syms y

equ = f(y) == 0;

ysol = solve(equ);

ysol = unique(ysol);

j=1;

for i= 1:size(ysol)

if ysol(i) <= ymax && ysol(i) >= ymin

yzero(j,1) = ysol(i);

j= j+1;

end

end

figure;

hold on;

for k= 1:size(yzero)

line([-1,1],[yzero(k),yzero(k)],'Color','black')

end

axis([-1, 1, ymin, ymax]);

values(1) = ymin;

for q=1:size(yzero)

values(q+1) = yzero(q);

end

values(length(values)+1) = ymax;

for w = 1:length(values)-1

test = values(w)+0.1;

sol = f(test);

if sol>0

plot(linspace(-1,1,100),linspace(values(w),values(w+1),100),'b.')

else

plot(linspace(-1,1,100),linspace(values(w),values(w+1),100),'r.')

end

end

hold off

end

f = @(y) ((y+4)^3)\*((y-2)^2)\*y;

phaseLine(f,-6,4)



**Question 5**

Part 1, alpha = -4



Where the equilibrium points are at

ysol =

0

-2i

2i

Where two points are imaginary so weren’t plotted properly in the phaseLine graph, since alpha was negative we got imaginary equilibrium points. y=0 is also an equilibrium point.

Part 2, alpha = 0



The equilibrium solution is at point y=0, where values below 0 are negative and above 0 are positive so since it’s – to + then y=0 is unstable equilibrium

Part 3, alpha = 1



The equilibrium solutions are at points y=-1, y=0, y=1

Where values below -1 are negative and above -1 are positive so since it’s – to + then y=-1 is unstable equilibrium

Values below 0 are positive and above 0 are negative so since it’s + to - then y=0 is stable equilibrium

Values below 1 are negative and above 1 are positive so since it’s – to + then y=1 is unstable equilibrium

Chart

Description automatically generated**Question 6**

Where are they on this new picture? From this picture, what can you say about the phase line at α = 4?

At alpha = -4 there is no equilibrium points except zero where we see a change as the others are imaginary. At alpha =0 the equilibrium points are at y=0 where the change is also from- to +. At alpha = 1 the equilibrium points are at y=-1,y=0,y=1 and the changes are similar as the phaseLine for y=-1 – to +, for y=0 + to -, for y=1 – to +. The equilibrium points are located at the black line.

At alpha=4 from the graph the equilibrium points are at y=-2,y=0,y=2, where at y=-2 it’s – to +, y=0 is + to -, y=2 is – to +.

**Question 7**

Chart

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Where is the bifurcation point, and what changes there? You should think about where the equilibrium solutions are and their respective stability.

The bifurcation point is at alpha=1, the graph changes there as the equilibrium points are all located on the positive side of y instead of the negative side. For alpha values below 1 the equilibrium points are negative values of y and they change from + to – so they are stable. While alpha above 1 have positive values of equilibrium points and they change from – to + so they are unstable. y=0 is always an equilibrium point and for values of alpha below 1 it’s unstable while alpha above 1 y=0 is stable. At alpha=1, y=0 is semi stable.

Chart

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